Review Sheet for Midterm II

Math 20: Introduction to Linear Algebra and Multivariable Calculus

November 17, 2004

Review Materials and Problems

The midterm covers Sections 3.1, 3.2, 4.9, 5.1–5.3, 6.1, 6.3–6.5 of Lay. Start by making sure you know how to do all of your homework problems. Solutions are posted on the course web site. You can also try additional, unassigned problems from any section you don't understand completely.

There is a set of supplementary problems after each Chapter in Lay. The answers to the odd problems are in the back of the book.

- For Chapter 3, don't worry about problems 13–15, 18–20
- None of the supplementary problems in Chapter 4 are germane
- For Chapter 5, skip 24–26
- For Chapter 6, skip 15–20

Review Topics

These are things you should know how to do in preparing for the second exam, organized by topic.

Introduction to Determinants (Section 3.1 of Lay)

- Compute the determinant of any square matrix by cofactor expansion

Properties of Determinants (Section 3.2)

- Compute the determinant of a square matrix using row operations
- Apply the properties $\det(AB) = \det(A)\det(B)$ and $\det(A^T) = \det(A)$

Eigenvectors and Eigenvalues (Section 5.1)

- Verify that a given number or vector is an eigenvalue or eigenvector of a given matrix
- Given a matrix and an eigenvalue, find a basis (linearly independent set of spanning vectors) for the corresponding eigenspace.

The Characteristic Equation (Section 5.2)

- Find the characteristic polynomial of a square matrix
- Find the eigenvalues of a matrix

Diagonalization (Section 5.3)

- Given a square matrix, find its complete eigensystem (all eigenvalues and eigenvectors)
- Given a square matrix A, find an invertible matrix P and diagonal matrix D such that $A = PDP^{-1}$

Application to Markov Chains (Section 4.9)

- Use a Markov chain to model a dynamical system
- Find the steady-state vector of a Markov chain and make conclusions about the model

Inner Product, Length, and Orthogonality (Section 6.1)

- Compute the dot product of any two vectors in $\mathbf{R}^n$
- Decide whether two vectors are orthogonal

Orthogonal Projections (Section 6.3)

- Given a subspace W with orthogonal basis $\{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$, and $\mathbf{y}$ any other vector, decompose $\mathbf{y}$ into $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$, where $\hat{\mathbf{y}} \in W$ and $\mathbf{z} \in W^{\perp}$.
- Interpret the orthogonal projection of a vector onto a subspace as multiplication by a certain matrix (Theorem 10)

The Gram-Schmidt Process (Section 6.4)

- Given a subspace W with basis $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$, construct an orthonormal basis
- Given a matrix A, find an factorization $A=QR$, where Q has orthonormal columns and R is upper-triangular with positive entries on its diagonal.

Least-Squares Problems (Section 6.5)

- Find the set of least-squares solutions to an inconsistent system of linear equations $A\mathbf{x} = \mathbf{b}$.
- Find the least-squares error in such a problem.